High-dimensional sensitivity analysis applied at vehicle component and system level in the context of CO$_2$ exhaust emissions

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Abstract

This paper demonstrates how the validation and verification phase of prototype development can be simplified through the application of the Model Development Suite (MoDS) software by integrating advanced statistical and numerical techniques. The authors have developed and present new numerical and software integration methods to support a) automated model parameter estimation (model calibration) with respect to experimental data and, b) automated global sensitivity analysis through using a High Dimensional Model Representation (HDMR).

These methods are demonstrated at 1) a component level by performing systematic parameter estimation of various friction models for heavy-duty IC engine applications, 2) at a sub-component level by performing a parameter estimation for an engine performance model, and 3) at a system level for evaluating fuel efficiency losses (and CO$_2$ sources) in a vehicle model over 160 ‘real-world’ and legislated drive cycles.

Introduction

The explosion of Computation Aided Engineering (CAE) over the last 20 years has enabled engineers to make a greater number of design decisions up front at their desks and thus yielded lower cost, higher quality and more advanced technology than could have been achieved otherwise. In line with this, the modern development cycle has evolved toward the so called “V-model”. A summary of this process is presented in Figure 1.

Those tasks on the left-hand-side of the diagram involve desk-based design, simulation and analysis with corresponding tasks on the right related to physical testing of the technology. The technology is considered through simulation at a system level first with an increasing granular focus toward subsystems and finally at the component level. Once the prototype is built, systematic testing is performed at each level and any important data generated during these tests are used to validate the original simulations and to support further design iterations and improvements.

Figure 1: Standard “V-model” for prototype development

As vehicle technologies have become increasingly complex, the validation phase has become a multi-dimensional challenge involving many interdependencies, software tools and data sources. The current work focuses on supporting engineers (operating at all levels) through this validation phase by integrating advanced software, statistical and numerical techniques into the design process.

A high level summary of the types of task performed at this stage include:

1. Design optimization to achieve specific design constraints or targets i.e. improving the design.
2. Sensitivity and uncertainty analysis i.e. understanding and quantifying design improvements and their potential impact.
3. Model training, model calibration or parameter estimation, i.e. fitting models to data.
4. Generating surrogate models for application at a system or sub-system level i.e. integrating data (experimental or simulation) to represent a sub-system or component.

With human resources/hour being $10^2$ - $10^3$ times more expensive than computational resources/hour and access to high performance computing resources the norm in many organizations, it is favorable to automate the above tasks and run them as batch jobs and post-processing the analysis.
This paper seeks to build upon previous papers by the authors related to data storage, parameter estimation and uncertainty analysis in physics-based model development in component level automotive applications [2] [3] [4]. These works are further extended by demonstrating how the above high-level tasks can be supported by applying the Model Development Suite (MoDS) to applications at a component, sub-system and system level. The work flow is demonstrated through two new representative example case studies. The first example represents the end user as a correlation builder, i.e. they have some experimental data and wish to understand the data better by using non-dimensional analysis or by developing a single expression or equation for their data (for interpolation/extrapolation). In this example, we work at a component level and use engine friction loss correlations and use friction data provided by Caterpillar Inc. from a variety of engine sizes and designs.

The second example represents a typical application of a sub-system level analysis and how its design impact on the overall system design. The case study examines how engine (sub-component) design constraints impact on the vehicle fuel efficiency. This is achieved by performing a parameter estimation (to best fit all known experimental data) on the engine model and to carry out a HDMR analysis of the system. This analysis will be used to identify the key design parameters and their interdependencies.

Software and numerical method development

In this study the Model Development Suite (MoDS), a software tool which can be integrated into any numerical model design and application process is employed as the underlying infrastructure for the delivery of the analysis. A summary of the available workflow and analysis has expanded significantly from previous analysis applications such as soot modelling [2] [6], combustion system optimization [3] [4] and biofuel plant analysis [7] and this workflow is outlined in Figure 2.

All the methods outlined in Figure 2 will not be summarized in this paper, however it is important to describe those most relevant to this paper. These are summarized in the following sections.

Parameter estimation

Since most models are parameterized in terms of unknown constants, and we often have experimental data, it is desirable to perform parameter estimation to fit the model response to the experimental data. More explicitly, let:

- the model in question be expressed as \( f(x, \theta) \) where \( x \) is some vector of inputs which we know the values of and \( \theta \) are the parameters which we wish to estimate.
the experimental data be expressed as \( y_1, y_2, y_3, \ldots, y_n \) where \( y_i \) was obtained at condition \( x = x_i \).

To perform the parameter estimation, an objective function must be chosen, which represents some notion of distance between the model and experiment. The general form of objective function that MoDS supports is

\[
\Phi(\theta) = \sum_{i=1}^{n} g(y_i - f(x_i, \theta), w_i)
\]

where the \( w_i \) are weights associated with \( y_i \) and \( g(z, w) \) is some function of your choice taking a residual \( z \) as the first argument, and a weight \( w \) as the second argument. In the special case of least squares, set \( g(z, w) = z^2 \) and in the case of weighted least squares set \( g(z, w) = wz^2 \).

The parameter estimation then is performed by minimizing the objective function \( \Phi(\theta) \) with respect to \( \theta \).

### High Dimensional Model Representation

Often, one is interested in gauging how sensitive a physical system is to its inputs – this is hugely important in assessing the importance of an input in models of that physical system. There are two different notions of sensitivity commonly used: local and global. In a local approach, one simply takes a model \( f(\theta) \) and computes/estimates its derivative \( \nabla f(\theta) \) at a single input vectorial point of interest \( \theta = (\theta_1, \ldots, \theta_m) \). This notion of sensitivity is only able to capture local information by definition. The global approach prefers the bigger picture, and asks how much \( f(\theta) \) varies as \( \theta \) varies over a global range.

One such global approach is called High Dimensional Model Representation (HDMR) [5]. HDMR is a form of response surface, or surrogate model. Its main feature is the decomposition of the model function \( f(\theta) \) into the following form:

\[
f(\theta) = f_0 + \sum_{i=1}^{m} f_i(\theta_i) + \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} f_{ij}(\theta_i, \theta_j) + \ldots + f_{1,2,\ldots,m}(\theta_1, \theta_2, \ldots, \theta_m)
\]

Obviously, this form is non-unique, but is useful in splitting up the model into terms which progressively increase in complexity starting with the constant 0th order term \( f_0 \), the 1st order terms \( f_i \) which only depend on one variable, all the way up to the \( m \)th order term. For practical applications, it is usual to truncate the decomposition so that the highest order terms are 2nd order ones. To make the decomposition unique in a useful way, consider the following:

- suppose the input \( \theta \) (uniformly) randomly varies in a hypercuboid \( H \). This implies that every term above varies randomly (except \( f_0 \) of course).
- impose the constraint (in the above decomposition) that all the terms except \( f_0 \) i.e. \( f_i(\theta_i), f_{ij}(\theta_i, \theta_j) \) etc., vary independently with mean zero.

One can now ask how much each term varies (i.e. how important they are). The natural choice is representing variability is using variance. Notice that the constraint of independence of terms automatically gives us an ANOVA-like variance decomposition:

\[
\text{Var}(f(\theta)) = \sum_{i=1}^{m} \text{Var}(f_i(\theta_i)) + \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \text{Var}(f_{ij}(\theta_i, \theta_j)) + \ldots + \text{Var}(f_{1,2,\ldots,m}(\theta_1, \theta_2, \ldots, \theta_m))
\]

The notion of sensitivity that HDMR defines is the proportion of the total variance \( \text{Var}(f(\theta)) \) that is contributed by each term. More precisely, the sensitivity of \( f(\theta) \) with respect to a 1st order term \( f_i(\theta_i) \) is defined as

\[
S_{i} = \frac{\text{Var}(f_i(\theta_i))}{\text{Var}(f(\theta))} = \frac{\int f_i^2(\theta) \, d\theta}{\int f^2(\theta) \, d\theta}
\]

and similar for higher order sensitivities. The intuition is that the sensitivity represents the relative importance of each term in the decomposition of \( f(\theta) \). Notice that this notion of sensitivity is global since we imagined that \( \theta \) randomly varied in a global hypercuboid.

### Engine Friction Model Case Study

Assessing engine performance parameters such as brake mean effective pressure (BMEP), friction mean effective pressure (FMEP) and specific fuel consumption (SFC) using virtual engineering toolkits is important during the design phase of any IC engine development program. Furthermore, by reducing the computational expense associated with detailed predictive modelling of these engine performance parameters, the overall design cycle can be made cost- and time-effective.

The aim of this work is to facilitate automated development of data-driven computational surrogates which incur low computational expense (evaluation times in ms) and quantify their predictive capability by comparison with high-fidelity, first generation detailed physics-based model data. Furthermore, the surrogates were desired to be applied over multiple IC engine configurations, thus presenting a significant challenge relating to the accuracy of the computational surrogate.

MoDS was applied to reduce the computational overhead by producing data-driven surrogate models, which are both cheap to evaluate yet, are sufficiently accurate for most usages. Using MoDS the available data on the main end and the big end bearing was classified into training data and test data. The surrogate models were calibrated using the training data and then their robustness and predictive capability were assessed by comparison with the test data. With the model and corresponding data read in, it is possible to use the wide array of algorithms available in MoDS, however, in this case, it was only necessary to apply general-purpose optimization routines to 'train' the surrogate models. The MoDS toolkit was combined with users’ analysis in terms of assessing the underlying 'correlations' between the several input variables.
Assessing underlying correlations

Figure 3 shows a matrix of plots containing inputs and output variables relevant to evaluating friction mean effective pressure (FMEP) in an IC engine context. For example, it can be observed that there is a linear relationship between the number of big end bearings and the number of cylinders in an IC engine configuration. Similarly, there are more complex relationships between other pairs of variables, for example, the engine speed and bearing dynamic viscosity. Such correlations (or lack thereof) were taken into account when formulating friction model terms or when modifying them.

The base friction model was adopted from Shayler’s [8] FMEP expression which uses the theory of similitude to relate the bearing geometries, speed and viscosity to friction.

Friction model, M1

\[
FMEP = A \left( \frac{TMEP}{\sqrt{1 - \epsilon^2}} \right) + 4\pi \frac{N_b}{N_{cyl}} \delta_{cyl} \tau_{Contact\_pressure} + B_1 \tau_{OH\_Mass} + B_3 \tau_{Mat\_Mass} + \text{(linear terms)}
\]

where:

- \( \epsilon \) is the eccentricity as a function of the modified Sommerfeld number \( \sigma \)
- \( \sigma \) is Sommerfeld number and used to characterize a bearing with its geometry, viscosity, rotational speed and load
- \( \tau \) are the various torques from different sources

\( TMEP \) is the total Mean Effective Pressure, and is expressed as

\[
TMEP = 4\pi \frac{N_b}{N_{cyl}} \delta_{cyl} \times 4\pi^2 r^3 N \frac{L_b}{2}
\]

where:

- \( r \) and \( c \) are the bearing radius [m] and clearance [m] respectively.
- \( L_b \) is the bearing stroke length [m]
- \( \mu \) is the bearing dynamic viscosity [Pa·s]

The tunable parameters of this model are \( A \) and \( B \).

Friction model, MoDS model

\[
FMEP = A \left( \frac{TMEP}{\sqrt{1 - \epsilon^2}} \right) + 4\pi \frac{N_b}{N_{cyl}} \delta_{cyl} \tau_{Contact\_pressure} + B_1 \tau_{OH\_Mass} + B_3 \tau_{Mat\_Mass} + \text{(linear terms)}
\]

This model formulation was derived from a physics-based model with some extra linear terms as a bias-correction. This has a total of seven tunable parameters.

Additional friction model expressions

As is typical in the model development process, further model improvements are sought by engineers which can be taken forward to improve the performance of the model with respect to observed or target datasets.

When a model takes the form similar to those outlined above, i.e. a correlation or an empirical fit, it is relatively easy to justify “trial and error” model development. This is the addition of/or arbitrary modification of terms and parameters, copying and pasting of sub-terms, groups of terms, numerators, denominators etc. to produce equally valid expressions or models.
With this in mind, a further set of models (M2-M7) representing typical "trial and error" model development were defined and for completeness their results are also analyzed in the sections which follow.

Robustness analysis

As mentioned earlier, a robustness analysis for each model was performed to quantify the model "fit" to the new data sets as they were added. To analyze this, the data was split into two sets – training and test data. The training data was used for training the model, then applied to the test data to assess the predictive capability of the model.

This procedure was repeated by randomly sampling (without replacement) the training and test sets from the existing data set and averaging how the robustness measure behaves for each model.

As depicted in Figure 4, the MoDS-formulated model performed best overall, based on the error measure used.

Figure 4: Total % error (over all points) versus the number of training data points. It can be seen that the MoDS model performs best overall.

Parameter estimation

The models were optimized with respect to their tunable parameters using MoDS. The strategy applied was based on assessing which model 'fitted' the experimental data with lowest error. The influence of different training/test set sizes were analyzed by carrying the same analysis out for alternative test sets.

Results and analysis

After performing the training/test data analysis, it was identified that M6 (modified) appeared to be the most robust and fitted reasonably well. A close competitor was the M7 model – however, this was expected given its corresponding large number of tunable parameters. Figure 5 provides the comparison between the FMEP values calculated by the various optimized models.

Figure 5: FMEP [kPa] versus data point index. The points are the high-fidelity model evaluations (these are the targets). Red line is M6, green line is optimized MoDS model, the blue line is M7.

Drive-cycle, Vehicle and Powertrain Model Case Study

In this case study the objective was to understand the interdependent relationships between eight design parameters which are typically under the control of engineers in the early stages of the design process and investigate their impact on fuel consumption. The example vehicle was selected to be a mid-size vehicle powered by a gasoline fuelled internal combustion engine. Vehicle performance will be investigated over multiple regulated and real world drive cycles.

Experimental data description

The experimental data obtained for use in this study were in the form of a) drive-cycle data used as an input to the vehicle model to obtain vehicle performance data across all these cycles, and b) full load engine performance data of engine power and load. These data are summarized in the following sections.

Drive-cycle data

A total of 160 cycles are included in the database covering all types of regulated and "real-world" driving cycles [9].

These cycles are all time dependent profiles of vehicle speed (Figure 6) which can all be summarized by a mean and standard deviation plot such as that presented in Figure 7. The database includes legislative (NEDC, FTP75 etc.), various
representative "real-world" (motorway, suburban, congestion, urban etc.), test and historical drive cycles.

An overview of the cycles included in the database are presented in Figure 7 in terms of mean cycle speed and standard deviation. The figure shows that those cycles representative of motorway driving are typically high speed with a lower standard deviation, urban and suburban driving cycles are typically at more intermediate speeds with a greater variance, whereas congestion cycles are at lower speeds.

![Figure 6. Sample set of drive-cycles included in the database](image)

**Figure 6.** Sample set of drive-cycles included in the database

![Figure 7. Summary of the applied drive-cycles, white + represent NEDC (upper) and FTP75 (lower) cycles [9]](image)

**Figure 7.** Summary of the applied drive-cycles, white + represent NEDC (upper) and FTP75 (lower) cycles [9]

**Vehicle and powertrain**

A summary of the test vehicle and powertrain used in this analysis is presented in Table 1.

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<table>
<thead>
<tr>
<th>Table 1. Test vehicle and powertrain specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass [kg]</td>
</tr>
<tr>
<td>Frontal Area [m²]</td>
</tr>
<tr>
<td>Wheel radius [m]</td>
</tr>
<tr>
<td>Drag coefficient [-]</td>
</tr>
<tr>
<td>Rolling resistance coefficient</td>
</tr>
<tr>
<td>Number of forward gears</td>
</tr>
<tr>
<td>Bore [mm]</td>
</tr>
<tr>
<td>Stroke [mm]</td>
</tr>
<tr>
<td>Compression ratio [-]</td>
</tr>
<tr>
<td>Number of strokes [-]</td>
</tr>
<tr>
<td>Fuel</td>
</tr>
<tr>
<td>Boosting</td>
</tr>
</tbody>
</table>

The full load torque and power of the engine are presented in Figure 8.

![Figure 8. Power and torque at full load vs. engine speed for experimental data (marks) best Sobol (solid lines) and subsequent best post-optimisation (broken lines) points](image)

**Figure 8.** Power and torque at full load vs. engine speed for experimental data (marks) best Sobol (solid lines) and subsequent best post-optimisation (broken lines) points

**Model descriptions**

The model applied includes sub-models for the powertrain and vehicle including drive train and driver "gear-shift" behavior. The vehicle is simulated over all of the drive-cycles included in the database. The section which follows summarizes these sub-models.

**Drive-cycle and vehicle sub-model**

The drive-cycle and vehicle sub-model applied here has been published previously in [10] but can be summarized as follows:

**Aerodynamic, rolling and grade vehicle resistances**

Vehicle resistances are simulated by considering the mass of the vehicle, drag co-efficient, frontal area, rolling friction, incline and required speed/acceleration. The power required to follow the drive-cycle is computed as a function of time.
**Gearbox, transmission and driver model**

The number of forward gears are a user input to model and the corresponding total (gearbox and transmission combined) gear ratios are determined using the method and gear ratio scaling laws outlined in [10].

The torque/speed demand on the engine is then determined by considering the required power and the driver’s gear shift behavior. In the current analysis, it is assumed that the driver will change up a gear at the moment they exceed 3000 RPM and change down a gear at 1000 RPM.

An example of the torque/speed demand for the NEDC for the test vehicle is presented in Figure 9. Here the dots represent the time at that particular load speed point. The size of the dot being larger if the engine has spent longer at this load-speed point.

As observed the shift strategy limits the majority of the operating window to a band between 1000 and 3000 RPM (only the highest gear will allow the IC engine to exceed 3000 RPM). A significant proportion of this drive-cycle is acceleration events and hence the weighting is toward the full load line. This is in contrast to drive-cycles such as the “real-world” congestion cycle presented in Figure 10, where the weighting is typically toward lower engine speeds and loads.

![Figure 9. Engine speed vs. torque for the NEDC. Size of the circles represents the total percentage of time at that point.](image)

Figure 9. Engine speed vs. torque for the NEDC. Size of the circles represents the total percentage of time at that point.

![Figure 10. Engine speed vs. torque for the real-world congestion cycle. Size of the circles represents the total percentage of time at that point.](image)

Figure 10. Engine speed vs. torque for the real-world congestion cycle. Size of the circles represents the total percentage of time at that point.

**Powertrain sub-model**

The powertrain sub-model is a mean-value model which is derived from [11] however other applications of this numerical approach have been presented elsewhere [12] [13] [14]. In this study, the relevant aspects of the sub-model are summarized as follows:

**Intake/exhaust network**

The duct systems are simulated as an orifice restriction, with the diameter of these systems, \(D_{int}\) and \(D_{exh}\) are considered model parameters representing the pressure losses caused by flow through the intake and exhaust systems respectively. In addition, a manifold temperature, \(T_{man}\) must be defined by the user.

**Volumetric efficiency**

The volumetric efficiency, \(\eta_{vol}\) is the main controlling term in obtaining the mass flow rate from the intake duct into the cylinders. The proposed model we apply the following correlation from [14].

\[
\eta_{vol} = C_{vol1}\sqrt{\frac{P_{int}}{RPM}} + C_{vol2}\sqrt{RPM} + C_{vol3}
\]

Where RPM is the engine rotational speed, \(P_{int}\) is the pressure downstream of the throttle in the intake system and \(C_{vol1}, C_{vol2}\) and \(C_{vol3}\) are model parameters.

**Friction**

In order to calculate an estimate for friction losses, the standard Chen-Flynn friction correlation model is applied [15].

\[
F_{\text{MEP}} = C_{CF1, B_{CF} P_{\max} + C_{CF2, 1.0E5} S_{p} + C_{CF3, 1.0E5} S_{p}^2}
\]

Where \(A_{CF}, B_{CF}, C_{CF}\) and \(Q_{CF}\) are user defined empirical parameters, \(P_{\max}\) is the peak in-cylinder pressure and \(S_{p}\) is the computed mean piston speed.

**Obtaining a load-speed performance map**

Fuel efficiency maps are produced by carrying out sweeps of engine speed and engine loading conditions. Obtaining such a map takes around five seconds on a standard desktop machine.

**Automated parameter estimation**

As declared above, the powertrain sub-model includes parameters which require estimation on the basis of experimental (or other) known data. In the present analysis, the data related to the powertrain were limited to those presented in Table 1 and in Figure 8. These data were selected as they represent the limited amount of data which can be obtained from the public domain on vehicles and their corresponding powertrain systems.
In order to build a model of the vehicle, firstly the model parameters for the powertrain were obtained using parameter estimation. All other numerical parameters were considered insensitive (and not included) and to minimize the number of active parameters in the analysis (and required computational time) these were fixed at a default value. The details of the parameters and their expected bounds are listed in Table 2. The bounds were defined based on experience of the model through application to other equivalent engines and typical values published elsewhere [14].

The targets of the optimization were set as those presented as experimental data in Figure 8.

**Results**

Firstly, a total of 2000 Sobol points [16] were generated, these were then analyzed in terms of the objective function. The top ten minimized points are presented in the parallel coordinate plot presented in Figure 11.

The best objective function result is presented as a black line and its corresponding “fit” to the experimental data is presented in Figure 8. Following on from this, these ten points were then used as starting points for a more localized parameter estimation using the Levenberg-Marquardt optimization routine [16].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dint</td>
<td>Intake duct diameter [m]</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Dexh</td>
<td>Exhaust duct diameter [m]</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Tman</td>
<td>Intake manifold temperature [K]</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>CVol1</td>
<td>Volumetric efficiency parameter 1</td>
<td>0.02</td>
<td>0.15</td>
</tr>
<tr>
<td>CVol2</td>
<td>Volumetric efficiency parameter 2</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>CVol3</td>
<td>Volumetric efficiency parameter 3</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The optimal point from this analysis is also presented in Figure 8, the corresponding optimized set of parameters was then employed throughout the following analysis.

**Drive-cycle analysis results**

The powertrain was then simulated and a fuel consumption map generated. This was then imported into the vehicle and drive-cycle sub-model and the vehicle was simulated over the whole database of drive-cycles. The mean fuel consumption (g/km) for these cycles are presented as the color of the marks in Figure 7.

Typically, as would be expected those cycles with worst fuel consumption rates proved to be those in slow moving start-stop or congestion cycles as these typically resulted in lower load operation. Steady-state motorway drive-cycles typically proved the best overall fuel efficient per km.

**HDMR analysis**

Next an analysis of various vehicle/engine designs was carried out to understand the impact of typical design parameters on drive-cycle fuel efficiency.

The design parameters listed in Table 3 were obtained from the test vehicle and the applied as initial bounds. The bounds were defined by considering data obtained from specifications of equivalent vehicles in the mid-sized vehicle classes. As typical, data for engine friction and volumetric efficiencies are not available in the public domain however typical ranges have been included.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Initial</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_r$</td>
<td>Compression ratio [-]</td>
<td>11</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>$D_e$</td>
<td>Engine cylinder bore [mm]</td>
<td>81</td>
<td>75</td>
<td>96</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Volumetric efficiency parameter</td>
<td>0.28</td>
<td>0.18</td>
<td>0.38</td>
</tr>
<tr>
<td>$A_{CF}$</td>
<td>Engine friction [bar]</td>
<td>0.5</td>
<td>0.45</td>
<td>0.55</td>
</tr>
<tr>
<td>$M_v$</td>
<td>Vehicle mass [kg]</td>
<td>1352</td>
<td>1200</td>
<td>1500</td>
</tr>
<tr>
<td>$A_f$</td>
<td>Vehicle frontal area [m$^2$]</td>
<td>1.91</td>
<td>1.76</td>
<td>1.95</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Vehicle aerodynamic drag coefficient [-]</td>
<td>0.35</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$f_{ein}$</td>
<td>Rolling friction coefficient [-]</td>
<td>0.013</td>
<td>0.01</td>
<td>0.015</td>
</tr>
</tbody>
</table>

These ranges represent the typical design space for a mid-sized vehicle with a 4-cylinder gasoline fuelled internal combustion engine.

The HDMR surface was generated by solving the model over 10,000 Sobol points [17] over the 8-dimensional space defined in Table 3. The HDMR surface was then fitted to this surface using a Levenberg-Marquardt optimization routine.
Results

As an example of the resulting HDMR surface, subsets of the surface are presented in Figure 12, Figure 13, Figure 14 and Figure 15 for the 8-dimensions. The color contours represent the total fuel consumption over the NEDC. The chart was generated at the local point defined by the initial values from Table 3, next a 100 by 100 grid over the two dimensions of interest was generated. Each chart was generated using the HDMR coefficients at these points.

![Figure 12. HDMR surface showing the dependency of engine cylinder bore and compression ratio on total fuel consumed during the NEDC.](image)

![Figure 13. HDMR surface showing the dependency of vehicle mass and rolling friction coefficient on total fuel consumed during the NEDC.](image)

Figure 14. HDMR surface showing the dependency of engine volumetric efficiency and engine friction parameter on total fuel consumed during the NEDC.

![Figure 15. HDMR surface showing the dependency of vehicle front surface area and aerodynamic drag on total fuel consumed during the NEDC.](image)

At the local points presented in Figure 12, Figure 13, Figure 14 and Figure 15, the following first-order observations can be made.

1. Increasing compression ratio reduces fuel consumption.
2. A larger engine bore results in a larger engine capacity, the smaller this capacity the lower the fuel consumption.
3. A lower rolling friction coefficient reduces fuel consumption.
4. A vehicle with lower mass reduces fuel consumption.
5. Improved engine volumetric efficiency reduces fuel consumption.
6. Reduced engine friction reduces fuel consumption.
7. Vehicles with smaller frontal surface areas have improved fuel efficiency.
8. Vehicles with reduced aerodynamic drag have improved fuel efficiency.

Whilst many of these observations are well known engineering and "industry-wide" trends (well known to vehicle and engine designers), their relative importance and interdependence are key considerations in the design process. Nevertheless these figures show evidence of second order effects, for example in Figure 14 some non-linear behavior is observed. In this case, it is expected that a higher volumetric efficiency is less favorable.
because it results in poorer fuel consumption efficiency at full load through higher peak pressures and friction losses.

Global sensitivity analysis

As noted above, the relative importance and interdependence of these design parameters are key considerations in the design process. In an effort to summarize the above multidimensional trends, the global sensitivity coefficients from the HDMR analysis are presented in Figure 16 and Figure 17 for the US-FTP75 and congestion drive cycles respectively.

Figure 16. Global sensitivities with respect to overall fuel consumption for the US-FTP75

Over the design space presented in in Table 3, these two charts show the relative importance of these design parameters on the overall fuel consumption rate. In the design space provided they show the following:

1. The importance of sizing the engine capacity (here represented by scaling the bore) is most important over both drive cycles.
2. In congestion the importance of aerodynamic design improvements make no significant impact.

Figure 17. Global sensitivities with respect to overall fuel consumption for the congestion cycle

3. In congestion, due to repetitive acceleration/braking the role of the vehicle mass is more important than in the US-FTP75 drive cycle.

Summary

The two case studies demonstrated:

1. Combining data and physics-based models is simplified through the workflow and application of the advanced statistical and numerical techniques in the Model Development Suite (MoDS).
2. By combining these techniques, a greater insight into the sensitivities and interdependency of a multidimensional design spaces can be achieved through a HDMR analysis.

References


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